Logic and Ontological Pluralism*

Jason Turner

November 1, 2010

According to *ontological pluralism*, there are different modes of being — different ways to exist. The view has been thought dead for a long time, destroyed by the Quinean doctrine that to be is to be needed as the value of a variable bound by an existential quantifier.

Announcements of its death were premature. Ontological pluralism is consistent with the Quinean doctrine if there are *multiple existential quantifiers*. We can have two modes of being — perhaps one for abstracta and another for concreta — and still stay broadly within the Quinean tradition if we also have two quantifiers, say '$\exists_a$' and '$\exists_c$' for abstracta and concreta, respectively. To be (in the abstract way) is to be needed as the value of a variable bound by '$\exists_a$'; to be (in the concrete way) is to be needed as the value of a variable bound by '$\exists_c$'.

A defense of this sort of pluralism requires movement on many fronts. In recent work, Kris McDaniel (2009, forthcoming) and I (2010) have developed the view, and defended it against several objections each of which, if correct, would show the view false. But a further objection has yet to be addressed: the objection that, although not *false*, ontological pluralism fails to be a distinct view from its apparent foe, ontological monism. According to this view, ontological pluralism is a mere *notational variant* to ontological monism. There is no battle here at all: just one army that can't agree on what to call itself.

I am here to respond to this further objection. More precisely, I will argue that, given a plausible thesis about the relationship between metaphysics and logic, we have good reason to think that ontological pluralism and ontological monism are not notational variants but rather genuine alternatives to each other. In §1, I state more precisely the theory to be defended and the objection it is to be defended from. In §2, I give the basic idea behind the argument. §3 contains a much-needed aside on some details involving the logic(s) of ontological pluralism. §4 provides a technical generalization of §2's argument, and §5 draws some conclusions from it.

*This paper grew out of an extended discussion with Jonathan Schaffer. So: Thanks, Jonathan! Thanks also to Robbie Williams and an anonymous referee for valuable suggestions and guidance.

1 Re-introducing Ontological Pluralism

1.1 Fundamental Languages

Metaphysical debates are often about reality's ultimate structure. Theorists who agree on the appearances disagree about how the world generates those appearances. They disagree about how the fabric of reality is stitched together. And since they tend to produce *theories*, they codify their disagreement as one about which sort of expressions latch on to reality's ultimate structural features — which 'carve reality at its joints', so to speak — and which ones don't.

Let's consider some examples. Some philosophers (e.g. Theodore Sider (2001: 11–25) and D. H. Mellor (1981, 1998)) deny that reality is fundamentally *tensed*. Talk about what was or what will be going on is, according to them, ultimately just talk about what (tenselessly) *is* going on at some time in the past or the future of the utterance. But other philosophers (e.g. A. N. Prior (1968) and Peter Ludlow (1999)) think instead that if you only talk about what goes on tenselessly at various times you miss out on important, tensed facts. Talk about what was or will be going on outstrips talk about what happens to be going on tenselessly at various times, they hold, because talk about what was or will be going on includes important additional information about which times *were* present, which times *will be* present, and which time *is* present *now*.

Similarly, some philosophers aim to reduce modality. Talk about what could or could not be the case is, according to them, ultimately just talk about what is or is not the case in some spatiotemporally disconnected spacetime (e.g. David Lewis (1986)) or reducible to some sort of linguistic convention (cf. Sider 2003: §4). But other philosophers (e.g. Prior (1977) and Alvin Plantinga (1987)) think instead that a metaphysical reduction of modality is a mistake. Whether or not such-and-so could be the case is, according to them, written in to the fabric of reality: modal talk latches on to some fundamental modal joint.

Let a *fundamental language* be a language where every (undefined) expression is supposed to 'carve reality at the joints' — to correspond to some ultimate structural feature of reality. And let a *fundamental theory* be a theory written in a fundamental language. Then the disagreement between those who reduce tense and those who refuse to can be captured as a disagreement about whether the fundamental language involves any tensed locutions — such as tense operators WAS and WILL, or a tensed 'now' predicate that applies to times — or not. And the disagreement between those who reduce modality and those who do not can be captured as a disagreement about whether the fundamental language involves any modal locations — such as modal possibility and necessity operators '$\Diamond$' and '$\Box$', or a modal 'actual' predicate that applies to worlds — or not.

1.2 *Ontological Pluralism*

Ontological pluralism is introduced against this background picture of metaphysics. The ontological pluralist is someone who thinks not just that there are different *kinds of things*, but also that there are different *ways to exist*.

Since Quine, philosophers have seemed reluctant to admit that there might be any serious view here. Zoltán Szabó nicely summarizes:

> The standard view nowadays is that we can adequately capture the meaning of sentences like 'There are *F*s', 'Some things are *F*s', or '*F*s exist' through existential quantification. As a result, not much credence is given to the idea that we must distinguish between different kinds or degrees of existence. (2003: 13)

But why think there is only one existential quantifier? If there were multiple existential quantifiers, then why wouldn't there be multiple ways of being, or kinds of existence — one for each of the quantifiers?

Of course, everyone involved admits we could find a way to describe reality using multiple existential quantifiers: we could just use quantifiers restricted in various ways. For instance, if we wanted quantifiers '$\exists_{\mathrm{a}}$' meant to range over abstracta and '$\exists_{\mathrm{c}}$' meant to range over concreta, we could just define them up out of a 'generic' quantifier '$\exists^*$' that ranges over both, as follows:

(1) $\ulcorner \exists_{\mathrm{a}} x\phi \urcorner =_{\mathrm{df.}} \ulcorner \exists^* x(x \text{ is abstract } \& \ \phi)\urcorner$

(2) $\ulcorner \exists_{\mathrm{c}} x\phi \urcorner =_{\mathrm{df.}} \ulcorner \exists^* x(x \text{ is concrete } \& \ \phi)\urcorner$

Likewise, if we started out with the abstract and concrete quantifiers, we could define up the generic quantifier, as well as predicates 'is concrete' and 'is abstract', with:

(3) $\ulcorner \exists^* x\phi \urcorner =_{\mathrm{df.}} \ulcorner \exists_{\mathrm{c}} x\phi \vee \exists_{\mathrm{a}} x\phi \urcorner$,

(4) $\ulcorner t \text{ is concrete}\urcorner =_{\mathrm{df.}} \ulcorner \exists_{\mathrm{c}} x(x = t)\urcorner$,

(5) $\ulcorner t \text{ is abstract}\urcorner =_{\mathrm{df.}} \ulcorner \exists_{\mathrm{a}} x(x = t)\urcorner$.

So there can be no fight about whether there *could* be a language with many quantifiers, or a language with one.

But the ontological pluralist thinks there is more to be said. She admits that, given one language, we can always cook up the other. But she thinks that the language which uses multiple quantifiers is *metaphysically better* than the language which uses just one. Just as the modalist thinks it is a mistake to define modal locutions by quantification over worlds, and the temporalist thinks it is a mistake to define tensed locutions by quantification over times, the ontological pluralist thinks it is a mistake to define '$\exists_c$' and '$\exists_a$' by (3). It is these 'restricted' quantifiers that carve reality at its joints, she holds, and this fact must be respected by our metaphysics.

1.3 *The Worry*

Now, for the worry. As a warm-up, consider two modalists: Nick and Polly. Both agree that the fundamental language should include modal operators. But Nick insists that the fundamental language ought to include the necessity operator '$\Box$', whereas Polly thinks it ought to include the possibility operator '$\Diamond$'.

Given the well-known interdefinability of these operators — $\ulcorner \Box\phi \urcorner$ can be defined as $\ulcorner \sim\Diamond\sim\phi \urcorner$, and *vice versa* — even committed modalists may look askance at Nick and Polly's debate. Do we really do a better job of limning the ultimate modal structure of reality, we might wonder, with one of these expressions than with the other?

A 'No' seems reasonable here. Someone who gives this answer will think that Nick and Polly's proposed fundamental theories are mere *notational variants* of each other.

Notational variance, in this sense, outstrips interdefinability. After all, many modalists — even those suspicious of the debate between Nick and Polly — might agree that there is a sense in which modal operators are 'interdefinable' with quantifications over possible worlds. These modalists might be happy to 'define' possible worlds as maximal possible propositions in the standard way, and go on to admit that we can now re-define the modal operators by quantifying over these worlds. But they insist that a language that takes the worlds as basic is more metaphysically misleading than one that takes the modal operators as basic.

The sense in which Nick and Polly's debate looks like one about notational variants is that there is an interdefinaibility *and it is hard to think of one language as being more metaphysically perspicuous than the other*. More precisely, call theories T_1 and T_2 *notational variants* of each other iff (i) T_1 can define some of T_2's primitive expressions in such a way that every theorem of T_2 is also a theorem of T_1; (ii) T_2 does the same thing for T_1; and (iii) the languages of T_1 and T_2 are equally metaphysically perspicuous and no less metaphysically perspicuous than any other. Nick and Polly's theories look suspiciously like notational variants in this sense.

The worry for ontological pluralism I want to consider here is that it and ontological monism are notational variants, in this sense. More precisely, it is that, for every ontologically plural theory T_{O}, there will be an ontologically monistic theory T_{M} that is a notational variant of T_{O}. As a result, ontological pluralism can't be any more metaphysically 'correct' than ontological monism, just as (we might think) it can be any more metaphysically 'correct' to describe the world in terms of '$\Box$' than it is in terms of '$\Diamond$'.

Clearly, any pluralist theory (with finitely many quantifiers) will have an ontologically monistic counterpart[1], and that each theory can define the quantifier expressions of the other along the lines of (1)–(5). Both theories will satisfy clauses (i) and (ii) in the definition of notational variance. So to block the worry, we have to show these pairs of theories won't satisfy clause (iii): we had better find a reason to think that the two theories aren't equally metaphysically perspicuous.

1.4 Notational Variance vs. Deflationary Metaphysics

Before going on, note that the present worry about pluralism and monism is not tied to worries about whether there is a unique way to 'carve up reality'. Philosophers such as Hilary Putnam (e.g. 1987*a*, 1987*b*) and Eli Hirsch (e.g. 2002, 2007) have insisted that there are different things we could mean by quantifier expressions, that none of these candidate meanings is in any way metaphysically privileged, and that as a result there could be theories that are notational variants of each other even though they have quantifiers that act as though they range over different (and different numbers of) objects.[2] Perhaps one theory divides up reality in such a way that whenever there are three mereologically simple objects, 'there are only three objects' comes out true, and another theory carves things up so that whenever there are three mereologically simple obejcts, 'there are seven objects' comes out true. According to Putnam and Hirsch's deflationary view of ontology, neither theory would be metaphysically privileged compared to the other; they would be notational variants. And, in some hard-to-define but easy-to-understand sense, if Hirsch and Putnam are right, there isn't really a fact of the matter about how many objects there are.

This sort of ontological deflationism needs to be addressed, but I'm not going to address it here.[3] The specific worry about ontological pluralism is meant to be orthogonal to general deflationism about ontology. The worry described above assumes that the monist and the pluralist agree about *everything* except the num-

[1]This isn't quite right; see §3.1 for discussion.

[2]See also Kim (1993: ix–x) and Sosa (1993: 619–625).

[3]Although see Dorr (2005), Eklund (2007), Fine (2001, 2005: 261–270), Hawthorne (2006), Hawthorne and Cortens (1995: 154–157), Lewis (1983, 1984), and Sider (2009, 2001: xvi–xxiv) for critical discussion.

ber of quantifiers in the fundamental language. In particular, they agree about *what there is*, in the generic ('$\exists^*$') sense. They simply disagree about whether talking about what there is in this generic way carves reality at the joints. And the worry is that their disagreement is just like a disagreement between two realists about modality, one of whom insists that it is necessity rather than possibility that carves reality at the joints, and the other who instead insists that it is possibility rather than necessity that carves reality at the joints.

We must keep these two worries separate. Even someone who thinks that there is a perfectly determinate fact of the matter as to what there is and how many of them there are — someone who thinks there is a unique, privileged way of parceling out reality into object-sized bites — may still balk at a real metaphysical difference between a pluralist theory and its monist counterpart. It is the worries of philosophers of this sort I hope to relieve here.

2 How Logical Realism Taught Me to Stop Worrying

2.1 *Logical Realism*

The relief I can give stems from a particular view about the role of the logic of a fundamental language. For lack of a better name, I will call it 'logical realism', although it should not be confused with other views that go by that moniker. It comprises two basic ideas.

The first is that logical consequence, as it relates sentences in fundamental languages, is not simply an accident of notation. The expressions of a fundamental language correspond to reality's most basic structural features: it's ultimate building-blocks, as it were. But for every set of building blocks, there are facts about how they do and do not fit together. The idea here is that the logic associated with a fundamental language is supposed to track these fitting-together facts. If ϕ is a logical consequence of Δ, where both are in a fundamental language, this corresponds to some metaphysically important feature of their respective contents: ϕ says something that comes in a sense 'automatically' from all of Δ, thanks to the way reality's ultimate structural features can and cannot fit together.

The second idea is that there is an objective fact of the matter about which contents are related in this metaphysically important way. If C and D are so related, then they are so independently of what language we express them in. Thus, if ϕ and Δ of one language express the same respective contents as ψ and Γ of another, then ϕ bears this metaphysically important relation to Δ if and only if ψ does likewise to Γ.

The conjunction of these theses does *not* entail that every sentences bearing this metaphysically important relation to some others is a logical consequence of them. It entails rather that, for every pair $\langle \phi, \Delta \rangle$ with this feature, if ψ and Γ are in a fundamental language and have the the same respective contents as ϕ and Δ, then ψ is a logical consequence of Γ.

A few points of clarification. First, logical realism (in this sense) could be — but need not be — a substantive view about what logical consequence *is*. Perhaps our pretheoretic concept of logical consequence is designed to track the important metaphysical relation described above; perhaps not. For all logical realism says, logical consequence might be material consequence under all uniform substitutions of non-logical constants, or material consequence under all re-interpretations of non-logical constants, or some particular kind of logical modality, or none of the above. The view holds only that logical consequence, whatever it is, is well-equipped to track this relation in a fundamental language.

Second, logical realism need not refrain from relativizing *logical consequence* both to languages and to choices of 'logical constants' within those languages. To see why not, consider again the case of Nick and Polly, and the sentences

(6) $\Box\phi \supset \phi$

(7) $\sim\Diamond\sim\phi \supset \phi$

To begin, note that, for either of them, whether or not these sentences count as logical truths (i.e., logical consequences of the empty set) may very well depend on whether or not '$\Box$' or '$\Diamond$' is a logical constant. But there is still a way to make sense of logical realism: it should be understood as saying that, if Nick and Polly's theories are notational variants, then for *corresponding choices* of logical constants, either both (6) and (7) will be logical truths, or neither will. Since choosing '$\Box$' as a logical constant of Nick's language corresponds to choosing '$\Diamond$' as a logical constant of Polly's, this condition hold: when the modal operators count as logical, both of (6) and (7) are logical truths; when we leave the modal operators out, neither are.

We said the relation tracked by logical consequence in a fundamental language was not relativized to languages — so how could it be relativized to expressions within that language? But the picture was this: the fundamental expressions latch onto different of reality's building blocks, and the logical consequences correspond to the arrangements of blocks that are 'automatic', as it were. If we leave the modal operators out of our reckoning of logical consequence, we are asking whether the arrangement corresponding to (6) is automatic thanks to its non-modal parts; if we add the operators in, we are asking whether it is automatic given its modal parts, too.

Also, notice that there is an almost trivial sense in which (6) and (7) can only be logical consequences relative to their respective languages. If the languages are taken to have no undefined expressions, (7) isn't even a sentence of Nick's language. So the question must be whether or not it is a logical truth relative to the logical system involving '$\Diamond$'. But that doesn't threaten the cogency of logical realism. Clearly, the thought is that, if (6) is a logical truth in the system governing '$\Box$', then (7) should be a logical truth in the system governing '$\Diamond$' instead.

2.2 *A Constraint*

If T_1 and T_2 are notational variants, they are so thanks to some recipe for defining some of T_1's expressions using T_2's terms, and vice versa. As a result, for every pair of notational variants, there will be a *translation* between the theories' respective languages that codifies the result of grinding through these definitions. In this case, if T_1 and T_2 are notational variants and t is the translation scheme that embodies this codification, say that T_1 and T_2 are *notational variants under t*.

The thesis of Logical Realism discussed above motivates the following constraint:

(LR) If T_1 and T_2 are notational variants under t, then t preserves logic,

in the sense that, if ϕ and Δ are in T_1's language, $\mathscr{L}_1$ is the logic associated with the language of T_1, and $\mathscr{L}_2$ is the logic associated with the language of T_2, then ϕ is a logical consequence of Δ according to $\mathscr{L}_1$ if and only ϕ's translation under t is a logical consequence of Δ's translation according to $\mathscr{L}_2$.[4]

Here's why. Part of the idea behind the notational variance charge is that both theories are somehow just saying the same thing. The translations under which the theories are notational variants are showing which sentences of one theory are making the same claims as which sentences of the other. But a sentence ϕ of one theory is a logical consequence (in that theory's associated system) of some others if and only if it bears the metaphysically important property discussed above to the others. Since having this property is a feature of content, and since

[4] I assume — or pretend, if you prefer — that the specification of a language also fixes a specific logical system that goes with it. There is at least one sense in which this is an idealization: even if a metaphysician offers up a fundamental theory with its associated language, there will often be open questions about which logical system is supposed to govern it. For instance, Nick would have to answer questions about whether he takes the logic of '$\Box$' to validate S4 axioms or not. If we wanted to be precise, we could think of a fundamental theory as an ordered triple consisting of a language, a logic over that language, and a set of theorems that comprise the non-logical content of the theory. I will not be so precise in the text, though.

ϕ and Δ are supposed to have the same contents as their translations, ϕ bears this relation to Δ if and only if its translation bears it to Δ's, and these translations are so related if and only if it's a logical consequence in its own associated system. The argument, of course, runs both ways, and we have (LR).

2.3 *Assuaging the Worry: A First Pass*

Suppose we have a pluralist theory T_P with quantifiers '$\exists_c$' and '$\exists_a$' for concreta and abstracta. The original thought was that, by using (1)–(2), we could construct a monist counterpart T_M to this pluralist theory. And thanks to these plus (3)–(5), there would be a translation function t under which T_P and T_M were notational variants.

The first pass argues that, given the (LR) constraint, the original thought was misguided. For consider the sentence of T_M:

(8) $\forall^*$x(x is concrete $\vee$ x is abstract)

This is not a logical truth; there is nothing in classical logic that requires everything to be either concrete or abstract. But t translates it into

(9) $\forall_1$x($\exists_1$y(x = y) $\vee$ $\exists_2$y(x = y)) & $\forall_2$x($\exists_1$y(x = y) $\vee$ $\exists_2$y(x = y)).

This is a logical truth — a theorem of pluralist logic. So t translates a non-logical truth of the monist's language into a logical truth of the pluralist's, in violation of (LR).

Some might think this is too fast. Suppose the monist defined 'x is abstract' as 'x is not concrete'. In this case the monist would would think of (8) as a derivative logical truth which reduces to

(10) $\forall^*$x(x is concrete $\vee$ x is not concrete)

Does this resurrect the worry?

Suppose that 'x is abstract' is indeed defined as 'x is not concrete'. Notice that now (1) becomes

(1′) $\ulcorner \exists_a x\phi \urcorner =_{\text{df.}} \ulcorner \exists^* x(x \text{ is not concrete } \& \ \phi)\urcorner$

Now consider the monist's

(11) $\sim\exists^*$x(x is concrete & x is not concrete).

This is a logical truth. But, under the (new) translation, it turns into

(12) $\sim[\exists_c x(\exists_c y(x = y) \ \& \ \exists_a y(x = y)) \vee \exists_a x(\exists_c y(x = y) \ \& \ \exists_a y(x = y))]$

which is logically equivalent (in the pluralist's system) to

(13) $\sim\exists_{c}x\exists_{a}y(x = y)$.

But (13) is *not* a logical truth — and so neither is (12) — which means we are still translating logical truths into non-logical truths and contravening (LR).

2.4 *An Objection, and a Preliminary Reply*

It's sometimes said that nothing could be both abstract and concrete, and so pluralist logic should be modified to rule this out, thus making (13) a logical truth after all. But I doubt that there is anything inherent in the idea that there are multiple modes of being which requires that these modes not overlap. Granted, we tend to think of ontological categories such as 'concrete' and 'abstract' as mutually exclusive. It is not obvious that they must be. (Mightn't space-time points be both concrete and abstract?) But even if they are, that doesn't mean all potential applications of ontological pluralism must follow suit. The abstract/concrete division is just a handy illustration for getting into the pluralist's mindset; there are other things we might want to use modes of being for.

On some readings of Descartes, for instance, his distinction between objective and formal reality is a distinction between two different kinds of being. To have objective reality is to have a certain mental mode of being; to have formal reality is to have a different, non-mental mode of being.[5]

This interpretation has it that one and the same thing can have both modes of being. When Descartes writes in the first set of replies

> ... the idea of the sun is the sun itself existing in the intellect — not of course formally existing, as it does in the heavens, but objectively existing, i.e. in the way in which objects normally are in the intellect, (1984: 75)

this interpretation takes him as saying that there is one thing — the sun — which has two modes of being: it has the 'objective', mental one insofar as it exists in a mind, and it has the 'formal', mind-independent one insofar as it exists 'in the heavens'. No Cartesian pluralist of this kind will be willing to say that modes of being cannot overlap.

If this is right, then at least many pluralists have reason to endorse a logic which does not make (13) a logical truth. Such pluralists can use the above argument to show that their theory is not a notational variant of its monist counterpart, at least not under the *t* considered here.

[5]Cf. Normore (1986: 235–238), Alanen (1994: 232–234), and Hoffman (1996: 368–371).

Of course, the argument doesn't help pluralists who do think (13) is a logical truth. And it doesn't show that there is *no* translation function under which a given pluralist theory even of the Cartesian kind is a notational variant of a monist one. But it provides a flavor for the kind of (LR)-based resistance to the notational variance worry, and a template for a more general resistance.

In §4, we'll extend the basic idea here to see just what it takes to find a logic-preserving translation function between pluralist and monist theories, and find that the cases are few and far between. First, though, we'll clear up a few issues about the logic of ontological pluralism.

3 Logic for Ontological Pluralists

3.1 *To Sort, or not to Sort*

Our ontological pluralist insists that the fundamental language uses multiple existential quantifiers. But there are a number of ways to incorporate multiple quantifiers into a language. One way is to have a *sorted* language: every predicate and term gets assigned a specified sort, and a formula is only well-formed if the right sorted of terms go into the right sorted slots.

So far, we have supposed that our pluralist opts for an *unsorted* language, which has no such restrictions. We had good reason: the worry raised in §1.3 is hard to state against an unsorted pluralist. The 'definitions' of the monist's $*$-quantifiers won't be well-formed in a sorted pluralist language. But the ill-formedness of (3) above isn't enough to show that no (sorted) pluralist theory and its monist counterpart are notational variants under some translation scheme; it faces the worry as well, although in a more subtle guise.

To my mind, both the sorted and the unsorted versions of pluralism are of interests, each with distinctive strengths and weaknesses. The sorted pluralist, for instance, gets to preserve the intuition that a sentence such as 'The number seven is red' isn't just false, but meaningless. On the other hand, the unsorted pluralist gets to preserve the intuition that one and the same metaphysical feature can apply to things of different ontological categories. For instance, a pluralist might think that the abstracta include sets, which can have both abstracta and concreta as members. She could help herself to two different set membership predicates, differing on their sortal restrictions; but as these are predicates of a fundamental language, that will reflect a metaphysical difference between the relation abstracta bear to sets and the relation concreta bears to sets — a difference she may reject. If so, she will want a single predicate in her fundamental language, and thus need a predicate with sorting restrictions limited.[6] Furthermore, any pluralist who wants to insist that one and the same thing can have two kinds of being will look askance at sorted pluralism; with a sorted identity predicate, it has no way to formulate sentences such as (13) or its denial.[7]

I consider both forms of pluralism here, so as not to play favorites. The sorted version of pluralism has a straightforward logic. Its axiom system is just that of classical first-order logic, except that the sorting restrictions limit what instances the axiom schemas can have.[8] A model for a sorted language is a pair $\langle d, I\rangle$ of a 'domain function' from (universal) quantifiers to sets of objects; intuitively, $d(\forall_i)$ can be thought of as the domain of $\forall_i$. The union of d's range is called the model's 'total domain'. I assigns names of sort i to $d(\forall_i)$, and sets of n-tuples of objects from the total domain to predicates Π, with the restriction that if $\langle o_1, \ldots, o_n\rangle \in I(\Pi)$, then for each $i = 1, \ldots, n$, if Πs i-th place is of sort j, then $o_i \in d(\forall_j)$.

Call this system *OPS* (for 'Ontologically Plural and Sorted'). Its behavior and properties are fairly-well known, and completeness theorems can be found in many introductory texts (see, e.g. Enderton 2001: 295–299). Logic for the unsorted pluralist, by contrast, needs a bit of finesse. We'll look at it in the next few sections.

3.2 *The 'There Can Be Only One' Argument*

A pluralist without sorting restrictions cannot endorse the classical inference rules:

> **Classical Universal Instantiation** (CUI): For any term t, $\forall_i x\phi \vdash \phi[t/x]$.
>
> **Classical Universal Generalization** (CUG): If $\phi_1, \ldots, \phi_n \vdash \psi$ and t does not occur free in $\phi_1, \ldots, \phi_n$, then $\phi_1, \ldots, \phi_n \vdash \forall_i x\psi[x/t]$.

If she did, she would face the 'There Can Be Only One' argument, which runs as follows.

Let t be a term that does not occur free in $\forall_c x\phi$. By CUI:

(14) $\forall_c x\phi \vdash \phi[t/x]$.

[6] Note blinded for review.

[7] We might also have a sorted pluralist language with a single unsorted identity predicate; such a language will face the same problems as the unsorted one. We'll consider such a view below.

[8] I assume that there is one quantifier for every sort, and a quantifier is allowed to bind a variable only of the same sort. Similar results can be achieved by leaving the quantifiers unsorted; I sort them only for continuity with the unsorted languages.

But since t does not occur free in $\forall_c x\phi$, CUG licenses

(15) $\forall_c x\phi \vdash \forall_a x(\phi[t/x])[x/t]$.

But $(\phi[t/x])[x/t]$ just is ϕ, so we get:

(16) $\forall_c x\phi \vdash \forall_a x\phi$.

The argument is perfectly general, so all of the '$\forall_i$'s are provably equivalent — whether they are indexed with 'c' and 'a' for concreta or abstracta, or in some other way.[9] But this makes it difficult to think of the various quantifiers as ranging over distinct domains, thereby threatening ontological pluralism's viability.

Unsorted pluralists can solve the problem by rejecting classical rules in favor of free-logic-style ones. In free logics, which allow for empty names, and inclusive logics, which allow for empty domains, we can only generalize from or instantiate to a term t if we have a premise that tells us that t exists, or (in a system with identity) that there is something that t is identical to. If instead we think that there are different ways in which a thing might exist, when we want to instantiate to t from a quantifier that represents a certain kind of being, or when we want to generalize from t to a quantifier that represents that kind of being, we ought to have a premise that tells us t denotes something with that kind of being. More precisely, for each possible indexing i of the quantifier, we have:

Pluralist Universal Instantiation (PUI): For any term t, $\exists_i y(y = t)$, $\forall_i x\phi \vdash \phi[t/x]$.

Pluralist Universal Generalization (PUG): If $\phi_1, \ldots, \phi_n \vdash \psi$ and t does not occur free in $\phi_1, \ldots, \phi_n$, then $\phi_1, \ldots, \phi_n$, $\exists_i y(y = t) \vdash \forall_i x\psi[x/t]$

The argument cannot be revived using these inference rules; the only way to get from $\forall_c x\phi$ to $\forall_a x\phi$ is to have as a premise $\exists_c y \exists_a z(y = z)$, which is not itself derivable from PUI and PUG.

3.3 *Unsorted Logic*

Here is one straightforward way to implement the above idea: we take an axiom system for a free logic and adapt it for use in a setting with multiple quantifiers.

[9] The argument seems to originate with Harris 1982, and versions of it can be found in Williamson 1988 and 2006, and McGee 2000 and 2006. I discuss it in Turner 2010: §5 where I give the response found below.

Let *FL* be a free-logical system — the the system $\text{PFL}_{2=}$ from Lambert 2001: 265, say. We define an *FL-model* as an ordered triple $\langle O, D, I \rangle$ of an 'outer domain', an 'inner domain', and an 'interpretation function'. Names get to be assigned by I to anything in either O or D, but the quantifiers range only over D; with that understood, truth relative to a model is defined as you would expect.

If there is a proof of ϕ from Δ in this deductive system, we write $\Delta \vdash_{\text{FL}} \phi$. And if every FL-model of Δ is an FL-model of ϕ, then we write $\Delta \models_{\text{FL}} \phi$. These relations have the advantage of satisfying

FL Completeness: $\Delta \vdash_{\text{FL}} \phi$ iff $\Delta \models_{\text{FL}} \phi$,

as has been shown by Leblanc and Thomasson (1982).

When we adapt *FL* for pluralist use, we get a system *OPF* (for 'Ontologically Plural and Free') that has as axioms every tautology and closed sentence of the form

(OP1) $\phi \supset \forall_i x\phi$[10]

(OPF2) $\forall_i x(\phi \supset \psi) \supset (\forall_i x\phi \supset \forall_i x\psi)$

(OPF3) $\forall_i y(\forall_i x\phi \supset \phi[y/x])$

(OPF4) $\forall_i x\phi[x/t]$ if ϕ is an axiom.

(OPF5) $t_1 = t_2 \supset (\phi \supset \phi[t_2//t_1])$

(OPF6) $t = t$

OPF has just one inference rule: modus ponens. If there is a proof of ϕ from Δ in OPF we write $\Delta \vdash_{\text{OPF}} \phi$.

A *OPF-model* is an ordered triple $\langle O, d, I \rangle$ of an outer domain, a domain function, and an interpretation function. The union of d's range is called the model's 'total inner domain'. I assigns names to members of either the outer domain or the total inner domain, and n-tuples of objects from the outer- and total inner domains to n-placed predicates.

A variable assignment on an OPF-model only assigns members of the total inner domain to variables. Truth of a well-formed formula on a model M, relative to a variable assignment g, follows the usual Tarskian recursive definitions, except that when it comes to the quantifier, the clause runs

[10]Notice that the indices are constant throughout; for each axiom, the same quantifier must be used in each instance.

($\forall_i$T) If $\phi = \forall_i x\phi$, it is true on M relative to g iff ϕ is true on M and g' for every g' that

(i) differs from g only on x (if at all), and

(ii) is such that $g'(x) \in d(\forall_i)$.

If every OPF-model of Δ is also an OPF-model of ϕ, we write $\Delta \models_{\text{OPF}} \phi$.

Given FL-Soundness and FL-Sompleteness, we can prove

OPF Completeness: $\Delta \vdash_{\text{OPF}} \phi$ iff $\Delta \models_{\text{OPF}} \phi$,

I will only sketch the proof here. First, we fix an ontologically plural language $\mathcal{L}_{\text{P}}$, with n universal quantifiers.[11] Then we let $\mathcal{L}_{\text{M}}$ be the ontologically monistic language gotten by replacing $\mathcal{L}_{\text{P}}$'s n quantifiers with a single quantifier, and adding n new one-placed predicates, 'Q_1', ..., 'Q_n'.

We pick out a very particular sentence of $\mathcal{L}_{\text{M}}$:

($\mathcal{Q}$) $\forall x(\text{Q}_1(x) \vee \ldots \vee \text{Q}_n(x))$

This says, in effect, that everything satisfies at least one of $\mathcal{L}_{\text{M}}$'s extra Q_i predicates. Next, we define a recursive 'translation function' T that takes us from sentences of $\mathcal{L}_{\text{P}}$ to sentences $\mathcal{L}_{\text{M}}$. The translation simply replaces each of the pluralists formulae $\ulcorner\forall_i x\phi\urcorner$ with the corresponding explicit restriction $\ulcorner\forall x(\text{Q}_i(x) \supset \phi)\urcorner$, and leaves everything else the same.

With these two tools in hand, we can prove:

Mutual Provability: $\Delta \vdash_{\text{OPF}} \phi$ iff $T(\Delta) \cup \{\mathcal{Q}\} \vdash_{\text{FL}} T(\phi)$.

Mutual Entailment: $\Delta \models_{\text{OPF}} \phi$ iff $T(\Delta) \cup \{\mathcal{Q}\} \models_{\text{FL}} T(\phi)$.

The proof of Mutual Provability essentially involves constructing FL-proofs from $\{\mathcal{Q}\}$ of all the translations of the OPF-axioms, and construct OPF-proofs of $\mathcal{Q}$ and the translations of the FL-axioms.[12] And the basic idea behind Mutual Entailment is that, for any OPF-model M of Δ, we can generate an FL-model of $T(\Delta) \cup \{\mathcal{Q}\}$, and vice versa, simply by swapping quantifier domains and 'Q_i' extensions. But I won't go into the details of the proofs of either of these lemmas here.

From Mutual Provability and Mutual Entailment, though, OPF Completeness follows. For $\Delta \vdash_{\text{OPF}} \phi$ iff $T(\Delta) \cup \{\mathcal{Q}\} \vdash_{\text{FL}} \phi$ (by Mutual Provability) iff $T(\Delta) \cup \{\mathcal{Q}\} \models_{\text{FL}} \phi$ (by FL Completeness) iff $\Delta \models_{\text{OPF}} \phi$ (by Mutual Entailment).

[11] I assume the language has only universal quantifiers in it; existential quantifiers are to be introduced by explicit definition.

[12] This gets a bit complex, because not every FL-axiom is the result of translating some sentence of $\mathcal{L}_{\text{P}}$; as a result, we have to introduce a 'back-translation' B and show that, for any ϕ in $\mathcal{L}_{\text{P}}$, ϕ and $T(B(\phi))$ mutually OPF-prove each other.

3.4 Alleviating Worries

3.4.1 Worry 1: Are These Models Kosher?

Some people may worry that these models — the ones for OPS and OPF — aren't the sort of things pluralists should be getting themselves involved in. Since they are defined up in an ontologically monistic metalanguage, the worry goes, the supposed 'ways of being' captured by a pluralist's multiple quantifiers are reduced to different *sets*, making these quantifiers mere restrictions of the 'bigger' metalanguage quantifier. Surely ontological pluralists were after something deeper than this!

This worry conflates a model theory for a language with an interpretational semantics that the ontological pluralist might accept for an ontologically plural language. Granted, OPS- and OPF-models are out of the question for interpreting *bona fide* uses of an ontologically plural language, for just the above reasons. But models in logic ostensibly have a different job: investigating the nature of logical consequence, given inferential constraints on logical terms. Not assigning meanings.

On some metalogical pictures, models investigate consequence by acting as potential interpretations of the language in question. On this picture, the models are all and only the possible assignments of meanings (or extensions) to non-logical expressions in the language. Model-theoretic entailment ($\vDash$) corresponds to logical consequence in the sense that, *whatever* meanings the non-logical expressions get, if the premises are true when so interpreted, then so are the conclusions. If this is our background metalogical framework, then it is important that one of the models be the *intended* model, and thus that the model theory provide a possible semantics for the language in question.

But on another picture, the job of models is (roughly) to correspond to logical possibilities — to states of affairs that aren't ruled out insofar as logic goes (cf. Etchemendy 1983, Field 1991). On this picture, we have a pre-theoretic grip on logical consequence. We use this grip to specify systems (inference rules and axioms) that we can tell are sound, that we can tell will never allow the deduction of ϕ from Δ when ϕ is not a consequence of Δ. And our pre-theoretic grip of consequence can guide us in our specification of models to the extent that, if Δ has a model, then we are confident that Δ is logically possible (and hence that if $\Delta \cup \{\sim\phi\}$ has a model, ϕ is not a logical consequence of Δ).

Since we build systems and models in this way, we guarantee that if $\Delta \vdash \phi$, then ϕ is a consequence of Δ, and we guarantee that if ϕ is a consequence of Δ, then $\Delta \vDash \phi$. Completeness proofs then show us that $\vdash$ and $\vDash$ are extensionally equivalent, and (by a 'squeezing' argument; cf. Kreisel 1967) that they are both extensionally equivalent to logical consequence.

To do this, models need only be just that — *models*, set theoretic pictures of how a world relates to a language. They need to represent the world well enough to show us why, if an argument has a countermodel, its conclusion is not a consequence of its premises. Clearly, though, models need not *be* ontologically plural in order to *represent* an ontologically plural world, any more than a painting of a fire needs to be warm to represent warmth.

3.4.2 Worry 2: Outer Domains?

A different worry lingers, at least for the unsorted pluralist. Both FL- and OPF-models use an outer domain — a domain of things that, according to the model, 'don't exist'. But even if we think models are only representations of ways the world could be (for all logic tells us), we might balk at this: there seems to be no part of the world for the outer domain to represent.

Fortunately, though, we can kick away the ladder of OPF-models' outer domains. Their presence makes the logic OPF 'free' in the sense that a name can have an interpretation not in the domain of any quantifier whatsoever. But pluralists need not commit to this option: they can introduce an axiom

(OP7) $\exists_1 x(x = t) \vee \ldots \vee \exists_n x(x = t)$

which guarantees, in effect, that every name gets assigned something in the domain of some quantifier or another. Call the resulting system *OPC* (for 'Ontologically Plural and (otherwise) Classical'), and let an *OPC-model* be just like an OPF-model except without an outer domain.

Now, where Γ is the set of all instances of (OP7), it is relatively easy to show

Mutual Provability 2: $\Delta \vdash_{\text{OPC}} \phi$ iff $\Delta \cup \Gamma \vdash_{\text{OPF}} \phi$.

Mutual Entailment 2: $\Delta \models_{\text{OPC}} \phi$ iff $\Delta \cup \Gamma \models_{\text{OPF}} \phi$.

(We get Mutual Provability 2 because adding a new axiom is like helping yourself to an (infinite) set of new premises. And we get Mutual Entailment 2 because any model of Γ will be one where no names get assigned to anything in the empty domain. Sentences with names that aren't assigned anything in the empty domain will have the same truth-value on any pair of models that differ only on the empty domain. So on models of Γ, we might as well scrap it.)

Of course, these two, along with OPF Completeness, give us

OPC Completeness: $\Delta \vdash_{\text{OPC}} \phi$ iff $\Delta \models_{\text{OPC}} \phi$.

And the logical consequence relation caught between this model- and proof-theory doesn't depend on any dodgy outer domains in the models.

3.5 *Other Business*

In OPC, names cannot be empty, but some domains can be: nothing requires that every domain have something assigned some name, and so (OP7) does not ensure that every domain be non-empty. OPC is thus *partially inclusive*: it can allow that some domains be empty, but not that all of them can.

The main obstacle to a completely inclusive variant of OPC is its reliance on names in the proof procedure. Any language $\mathcal{L}_\text{P}$ with names in it requires that the names have some assignment, and (OP7) forces that assignment to be ranged over by one of the '$\forall_i$'s. But we can modify the proof procedure to make the use of names a mere calculational device. If $\mathcal{L}_\text{P}$ is any unsorted pluralist language (even one without names) and S is any set of names, let $\mathcal{L}_\text{P}^S$ be the result adding the names in S to $\mathcal{L}_\text{P}$. Now, if ϕ and Δ are sentences of $\mathcal{L}_\text{P}$, let a proof of ϕ from Δ be any sequence of $\mathcal{L}_\text{P}^S$ (for some S) that has each entry as an axiom, a member of Δ, or following from earlier entires by modus ponens. Using techniques used to show completeness for name-involving proof systems, we can show that if there is a proof from Δ to ϕ in $\mathcal{L}_\text{P}^S$ and none of the names in S show up in Δ or ϕ, then if M is an OPC-model of the language $\mathcal{L}_\text{P}$, M is an OPC-model of Δ iff it is an OPC-model of ϕ.

This gives us a (name-using) proof theory for a name-free language, but it still won't make the logic completely inclusive. The problem now is that, even if ϕ and the sentences in Δ have no names in them, (OP7) will still be an available axiom (perhaps in an extended language) and can be used to derive

(17) $\exists_1(x = x) \vee \ldots \vee \exists_n(x = x)$.

The solution here is to replace the (OP7) axiom with an inference rule that allows us to write down instances of (OP7) when (and only when) t occurs in Δ. (17) can no longer be derived from name-free Δ. It is easy to check that the resulting completely free system is complete if the partially inclusive variant is.

Standard treatments of sorted first-order logic are fully exclusive: for each i, $\exists_i x(x = x)$ is a theorem. But it is relatively straightforward to build a system using the sorted equivalent of, e.g., the completely inclusive axiomatic system of Hailperin (1953), and prove it complete against a model theory like the one sketched in §3.1 that allows for models where $d(\forall_i)$ is empty for each i. If we want a partly inclusive variant, we add as an axiom

(18) $\exists_1(x = x) \vee \ldots \vee \exists_n(x = x)$

and throw out the model where every quantifier's domain is empty; if we want an exclusive variant, we change the '$\vee$'s in (18) to '&'s and throw out models where any quantifier's domain is empty.

Finally, as long as we're in the business of adding axioms: a pluralist who thinks that (as a matter of logic) the various modes of being cannot overlap can endorse a *unique sorting axiom*

(OPC8) $\ulcorner \forall_i \alpha \sim \exists_j \beta(\alpha = \beta)\urcorner$, where $i \neq j$,

to OPC and modify her model theory to insist that no object show up in multiple domains. Call the resulting system *OPC+*. With the help of

Mutual Provability 3: $\Delta \vdash_{\text{OPC+}} \phi$ iff $\Delta \cup \Gamma \vdash_{\text{OPC}} \phi$.

Mutual Entailment 3: $\Delta \models_{\text{OPC+}} \phi$ iff $\Delta \cup \Gamma \models_{\text{OPC}} \phi$.

where Γ is the set of all sentences of the form (OPC8), we can piggyback its completeness off that of OPC. We'll consider OPC+, along with OPC and OPS, in what follows.

4 Generalizing the Logical Realism Argument

If T_P is a pluralist theory, call it *notation fodder* iff there is a monist theory T_M and a translation function t between them that preserves logic. In this section, we prove that — given certain assumptions, which are rendered plausible by the nature of the notational variance worry we're grappling with here — in a fairly intuitive sense very few pluralist theories are notation fodder. In §5, I argue that this gives us reason to think that even those theories that are notation fodder are most likely not notational variants of any monist theory.

The result here is meant to apply to pluralists of all stripes — those who endorse OPC, OPC+, and OPS — although the form of the conclusion varies slightly depending on stripe. For portions of the argument where differences between these do not matter, I'll use 'OP' ambiguously for all three; when differences matter, I'll disambiguate.

4.1 Assumptions

Let T_{P} be a pluralist theory with $\mathcal{L}_{\text{P}}$ its language, and suppose it is notation fodder. Then there is a monist theory T_M and a logic-preserving translation scheme t[13] between them.

[13]Formally, we think of t as a pair of functions $\langle f, g\rangle$, where f takes us from sentences of $\mathcal{L}_{\text{M}}$ to sentences of $\mathcal{L}_{\text{P}}$, and g goes in the other direction. For simplcity, though, we'll just write application of either of these functions as '$t(\phi)$', letting context disambiguate which function is meant.

I assume that $\mathcal{L}_P$ and $\mathcal{L}_M$ have finite vocabularies. (I don't know any reason they would *have* to, but I take it that if only pluralist theories with infinite vocabularies are notation fodder, pluralism is in pretty good shape.) I assume further that both languages are at least partly inclusive and name-free. This is partly for mathematical tractability, and partly to stack the deck in favor of notational variance. The issue has to do with OPS: if it has a certain stock of names of different sorts, then certain logical possibilities the monist can recognize — such as the possibility of there being just one thing, which is named by all the names — won't correspond to any logical possibility the sorted pluralist recognizes. We might take this as a strike against the sorted theory being notation fodder; but we might also take it as an oddity of the way names are dealt with in first-order systems. Rather than stake too much on the issue, I prefer to leave names out of it altogether.

The logic of $\mathcal{L}_M$ we take to be classical (denoted by 'C'). Since C and all of the pluralist systems canvassed here are complete, we can identify logical with model-theoretic consequence (see §3.4.1). Thus the fact that t preserves logic entails:

Logical Realism: i) if Δ and ϕ in $\mathcal{L}_M$, then $\Delta \models_C \phi$ iff $t(\Delta) \models_{OP} t(\phi)$, and

ii) if ϕ and Δ in $\mathcal{L}_P$, then $\Delta \models_{OP} \phi$ iff $t(\Delta) \models_C t(\phi)$.

Furthermore, since t is a translation scheme, it should be

Recoverable: $t(t(\phi))$ is logically equivalent (in the relevant system) to ϕ.

The thought here is that translating between theories preserves, content, so when we translate ϕ into the other language and then back again, we should get something with the same content as — and thus logically equivalent to — ϕ.[14]

Note that that T_P's notation-fodderhood stems not from any general no-fact-of-the-matter-ism in ontology, but from a *specific* problem with ontological pluralism. We're allowed that there is a unique way to parcel out the world into object-sized bites; the notational variance worry is only skeptical about thinking the world cares which 'mode of being' those bites are assigned. This motivates a further assumption: since neither theory carves the world into different object-shaped bites than the other, they should both agree — and hence agree under the translation t — about how many things there (generically) are.

Models represent the world as having a certain number of object-shaped bites in it. This is represented in a C-model by its cardinality (the cardinality of its

[14]We might balk at this thought when it comes to names — 'a=b' might have the same content as 'a=a' without being logically equivalent — but since we're explicitly taking the theories to be name-free, this won't be an issue.

domain). Likewise, it is represented in an OPC or OPC+ model by that model's cardinality (the cardinality of its total domain). So for C and OPC/OPC+, the intuitive thought — that the translation should preserve how many things there (generically) are — tells us that if ϕ is only true on models of cardinality κ, $t(\phi)$ is likewise only true on models of cardinality κ.

But this intuitive thought is harder to make sense of for the sorted pluralist. Consider, for instance, an OPS-model M with just a in one domain, just b in another, and nothing anywhere else. In a certain sense, this is a 'two-membered' model. But, thanks to the sorting restrictions, this model is isomorphic to the intuitively one-membered model M' with a in both domains and showing up in every predicate extension where b showed up before. (Two models are *isomorphisms* if and only if no sentence of the modeled language is true on one and false on the other.) What should the sorted pluralist say about the respective universe sizes represented by M and M'?

First option: say that the question is ill-formed. The pluralist's fundamental language *can't make sense* of the question as to whether the thing ranged over by one quantifier is the same as the thing ranged over by the other; it can't make sense of cross-categorical counting sentences generally. So she might just reject the question. But then we already have strong philosophical reason to claim that her view is not a notational variant of the monists': he can ask a question she cannot make sense of. As far as I can see, this position is philosophically viable, and perhaps fits best with the motivations for sorted pluralism. But it scuppers the notational variance worry completely. So we can set it aside.

Second option: say that M represents the universe as having two things, and that M' represents the universe as having only one. Since M and M' are isomorphic, this is tantamount to admitting there is a metaphysically interesting question *the answer to which cannot be expressed in the fundamental language*. This does not look philosophically viable: the fundamental language is supposed to be the language used for saying everything of metaphysical interest that can be said. Let's set it aside, too.

Third option: say that both M and M' represent the universe as having two things in it. This view opts for maximal exclusion: categories never share members. Things in different domains are always treated as distinct; when the same thing shows up in different domains, this is seen as a model-theoretic abnormality, where the same thing is allowed to do double-representational duty thanks to the language's sorting restrictions. Call this the *exclusive option*. After the first option, it is probably closest to the metaphysical picture best supported by sorted pluralism.

Fourth option: say that both M and M' represent the universe as having one thing in it. This view opts for maximal inclusion: categories overlap insofar as they can. (In a model with a and b in one domain and c in another, there will be a further question as to whether c is identical to a or b; but for the purposes of counting, this question needs not be answered, and the pluralist might even refuse to make sense of it while still making sense of the counting question.) Call this the *inclusive option*; while perhaps not very conducive to the pluralist's motivations, we'll consider it here, too.

Fifth option: add an unsorted identity predicate to the otherwise sorted language. M and M' are no longer isomorphic to each other, and the plurallist can now say the cardinality of the model represents the size of the universe. Call this the *concessive* option.

So we now have four interpretations of the *size* of a model: the size of classical, OPF-, OPF+-, and OPS-models on the concessive option are their cardinalities; the size of an OPS-model taken exclusively is the highest cardinality of any model it is isomorphic to; and the size of an OPS-model taken inclusively is the lowest cardinality of any model it is isomorphic to. The intuitive thought from above now underwrites the assumption:

Equinumerosity: If a sentence ϕ is true only on models of size κ, $t(\phi)$ is true only on models of size κ also.

We'll disambiguate 'size' later, when required.

Before going on, two useful results and one useful bit of notation. First, since t is recoverable and preserves logic, it is also

Truth-functionally Conservative: For the relevant systems, (i) $t(\sim\phi)$ is logically equivalent to $\sim t(\phi)$, and (ii) $t(\phi \,\&\, \psi)$ is logically equivalent to $t(\phi) \,\&\, t(\psi)$.

(Of course, these will straightforwardly entail similar clauses for the other truth-functional connectives.) Letting '$\models$' represent consequence in the relevant system"

Proof of (i), left-to-right: $\{\phi, \sim\phi\} \models \bot$, so $\{t(\phi), t(\sim\psi)\} \models \bot$, so $t(\sim\phi) \models \sim t(\phi)$.

Right-to-left: suppose for reductio that $\sim t(\phi) \not\models t(\sim\phi)$. Then $\{\sim t(\phi), \sim t(\sim\phi))\} \not\models \bot$. So $\{t(\sim t(\phi)), t(\sim t(\sim\phi))\} \not\models \bot$. But by the left-to-right direction just shown, $\{\sim t(t(\phi)), \sim t(t(\sim\phi))\} \not\models \bot$, and so — substituting equivalents via recoverability — $\{\sim\phi, \sim\sim\phi\} \not\models \bot$. Contradiction.[15] QED

[15]This is the only place in the proof recoverability is assumed. In a multiple-conclusion setting (cf. Restall 2005) it isn't needed at all; we argue $\top \models \{\phi, \sim\phi\}$, so $\top \models \{t(\phi), t(\sim\phi)\}$, so $\sim t(\phi) \models t(\sim\phi)$. Thanks here to Robbie Williams.

Proof of (ii): since $\phi \ \& \ \psi \models \phi$, $t(\phi \ \& \ \psi) \models t(\phi)$. Similarly, $t(\phi \ \& \ \psi) \models t(\psi)$. So $t(\phi \ \& \ \psi) \models t(\phi) \ \& \ t(\psi)$. Conversely, $\{\phi, \psi\} \models \phi \ \& \ \psi$, so $\{t(\phi), t(\psi)\} \models t(\phi \ \& \ \psi)$, so $t(\phi) \ \& \ t(\psi) \models t(\phi \ \& \ \psi)$. QED

Next, if ϕ is a sentence of $\mathcal{L}_P$ (or $\mathcal{L}_M$), let $[\phi]$ be the set of OP- (or C-)models on which it is true. Here are two facts to keep in mind:

Fact 1: $[\ulcorner\phi \ \& \ \psi\urcorner] = [\phi] \cap [\psi]$

Fact 2: $[\phi] = [\psi]$ iff ϕ and ψ are logically equivalent.

Proof of Fact 1: $m \in [\ulcorner\phi \ \& \ \psi\urcorner]$ iff $m \models \phi \ \& \ \psi$ iff $m \models \phi$ and $m \models \psi$ iff $m \in [\phi]$ and $m \in [\psi]$. QED

Proof of Fact 2: $[\phi] \neq [\psi]$ iff for some model m, $m \in [\phi]$ and $m \notin [\psi]$ (or *vice versa*) iff $m \models \phi$ and $m \not\models \psi$ (or *vice versa*) iff $m \not\models \ulcorner\phi \equiv \psi\urcorner$ iff ϕ and ψ are not logically equivalent. QED

4.2 *The Generalization*

T_P is notation fodder, or so we are assuming. We will show that this means $\mathcal{L}_P$ meets some pretty specific constraints. When the constraints are narrow enough (as they are when the background logic is OPC), the argument comprises a reductio of the assumption, and therefore a reductio of the thought that T_p is a notational variant of a monistic theory. In the other cases, further argument (§5) supports this second conclusion.

Call a model *singular* iff it has a size of one. The argument proceeds first by showing that the existence of a translation t with the assumed properties forces a one-to-one function from certain sets of singular OP-models to certain sets of singular C-models. There are thus just as many sets of the former kind as sets of the latter kind. Then we show that considerations of how many sets of each kind there are mean that $\mathcal{L}_P$ must meet some fairly specific constraints.

4.2.1 The Correspondence

Recall that two models are isomorphisms if and only if no sentence of the modeled language is true on one and false on the other. Isomorphism is an equivalence relation, so the models defined by a given model theory can be divided into *isomorphism classes* — classes of all and only the models that are isomorphisms to each other.

Ultimately, we want to show that there is a one-to-one correspondence between isomorphism classes of singular OP-models and isomorphism classes of singular C-models. But we'll start by just showing that there is a one-to-one correspondence just between classes of singular models first. Then we'll show that a sentence ϕ characterizes a singular model up to isomorphism iff $t(\phi)$ does so as well. As a result, the correspondence we started out with, when restricted to isomorphism classes, generates the correspondence we ultimately want.

To begin, call a sentence *1-categorical* iff it is only true on singular models. For example, $\exists x \forall y(x = y)$ is a 1-categorical sentence of $\mathcal{L}_C$, as is anything that implies it; and $\exists_i x(\forall_1 y(x = y) \ \& \ \ldots \ \& \ \forall_n y(x = y))$ and its implicands are 1-categorical sentences of $\mathcal{L}_P$.[16]

Let $\mathscr{P}$ be the family of sets $[\phi]$ for 1-categorical ϕ in $\mathcal{L}_P$, and let $\mathscr{M}$ be the family of sets $[\phi]$ for 1-categorical ϕ in $\mathcal{L}_M$. Let F be the function from $\mathscr{P}$ such that $F[\phi] = [t(\phi)]$. By Equinumerosity, $F[\phi]$ will be in $\mathscr{M}$. Now we need to prove:

Claim 1: F is a one-to-one correspondence between $\mathscr{P}$ and $\mathscr{M}$.

We need to show that F is one-to-one (distinct members of $\mathscr{P}$ are mapped to distinct members of $\mathscr{M}$) and that it maps $\mathscr{P}$ onto $\mathscr{M}$ (every member of $\mathscr{M}$ is the value of T for some member of $\mathscr{P}$).

First: F maps $\mathscr{P}$ onto $\mathscr{M}$. Suppose ϕ is in $\mathscr{M}$; we need to show that, for some ψ in $\mathscr{P}$, $F[\psi] = [\phi]$. Note that $t(t(\phi))$ is logically equivalent to ϕ by Recoverability; so, by Fact 2, $[t(t(\phi))] = [\phi]$. But $[t(t(\phi))] = F[t(\phi)]$, and $t(\phi)$ is in $\mathscr{P}$, so we're done.

Second: F is one-to-one. Suppose $[\phi] \neq [\psi]$; then ϕ is not logically equivalent to ψ by Fact 2. In this case, $t(\phi)$ and $t(\psi)$ are not logically equivalent to each other either. So, by Fact 2, $[t(\phi)] \neq [t(\psi)]$. Since $F[\phi] = [t(\phi)]$ and $F[\psi] = [t(\psi)]$, this shows F is one-to-one. QED

So F is a one-to-one correspondence between sets of singular models. But not all of these sets will be isomorphism classes of models. $[\exists x \forall y(x = y)]$, for instance, is in $\mathscr{M}$, but its members are not all isomorphisms of each other.

We need a way to characterize isomorphism classes. If S is a set of models, call it *minimal* iff:

(i) S is not empty;

(ii) for some ϕ in the relevant language, $S = [\phi]$, and

(iii) for every ψ in that language, if $\psi \subset S$, then $[\psi] = \emptyset$.

[16]Different sentences will count as 1-categorical on OPS depending on whether we're being inclusive or exclusive, of course. That won't matter in the argument to follow; nothing about the definition of 'singular' will matter until §4.2.2, when we will disambiguate.

Minimal singular models will be the isomorphism classes we need. Since the languages have only finitary resources, every isomorphism class of singular models can be characterized by some sentence of the relevant language. Let ϕ be such a sentence; then it is minimal. If it weren't, there would be some ψ such that $[\psi]$ is non-empty and $[\psi] \subset [\phi]$. But then there would be a sentence, $\ulcorner \phi \,\&\, \psi \urcorner$, that differs in truth-value across isomorphic models, contra the definition of isomorphism.

Here is a useful fact about minimal sets of singular models:

Fact 3: If $[\phi]$ is a minimal set of singular models, then for any ψ, either $[\phi \,\&\, \psi] = [\phi]$ or $[\phi \,\&\, \psi] = \emptyset$.

> Proof of Fact 3: let $[\phi]$ be a minimal set of singular models. By Fact 1, $[\phi \,\&\, \psi] = [\phi] \cap [\psi]$, so $[\phi \,\&\, \psi] \subseteq [\phi]$. But, since $[\phi]$ is minimal, if $[\phi \,\&\, \psi] \neq [\phi]$, then $[\phi] = \emptyset$ (by condition (iii) in the definition of minimality). QED

We'll use this fact in proving

Claim 2: $[\phi]$ is a minimal set of singular models iff $[t(\phi)]$ is.

From Claims 1 and 2 combined, it follows that F, when restricted to the minimal sets in $\mathscr{P}$, is in one-to-one correspondence with the minimal sets of $\mathscr{M}$. Or, in other words, there is a one-to-one correspondence between isomorphism classes of singular models.

Proof of Claim 2: Suppose that $[\phi]$ is a minimal set of singular models. Then $\sim\phi$ is not a logical truth, in which case $\sim t(\phi)$ isn't, either, which means that $[t(\phi)]$ is not empty. So condition one is met. And condition two is met, since $t(\phi)$ is a sentence of the relevant language. So we need only to show that condition three is met, too.

Suppose otherwise. Then for some ψ, in the relevant language $[\psi] \subset [t(\phi)]$ and $[\psi]$ is non-empty. Since $[\psi]$ is non-empty and $[t(\phi)]$ is a set of singular models, $[\psi]$ is also a set of singular models, and so ψ is 1-categorical. Furthermore, $\ulcorner \psi \supset t(\phi) \urcorner$ is a logical truth, because every model of ψ is a model of $t(\phi)$.

In that case, there is a χ in the same language as ϕ which is 1-categorical and for which $t(\chi) = \psi$. But then $\ulcorner \psi \supset t(\phi) \urcorner$ just is $\ulcorner t(\chi) \supset t(\phi) \urcorner$, which is equivalent to $\ulcorner t(\chi \supset \phi) \urcorner$ by Conservatism. And this means that $\ulcorner \chi \supset \phi \urcorner$ is a logical truth, which means $[\chi] \subseteq [\phi]$.

Thus, by Fact 3, either $[\chi] = [\phi]$ or $[\chi] = \emptyset$. If $[\chi] = \emptyset$, then χ is a logical falsehood, which means that $t(\chi) = \psi$ is, too. But that would make $[\psi]$ empty, which it isn't. If instead $[\chi] = [\phi]$, then $[t(\chi)] = [\psi] = [\phi]$, in which case

$[\psi] \not\subset [\phi]$. Either way, we get a contradiction; so there is no ψ in the relevant language where $[\psi]$ is not empty and $[\psi] \subset [t(\phi)]$. So $[t(\phi)]$ meets condition three as well.

This proves the right-to-left half of the biconditional: if $[\phi]$ is a minimal set of singular models, so is $[t(\phi)]$. The converse follows when we substitute $t(\phi)$ for ϕ. This gets us that if $[t(\phi)]$ is a set of minimal models, then $[t(t(\phi))]$ is, too. But by Recoverability and Conservatism, $[t(t(\phi))] = [\phi]$. QED

4.2.2 Counting Models

So there is a one-to-one correspondence between isomorphism classes for singular models of the respective model theories — there are just as many isomorphism classes of singular OP-models as there are of singular C-models.

But we can figure out just how many isomorphism classes each model theory has. There will be one isomorphism class for each way to build a model of size one for each logical system (and disambiguation of 'size').

Let's begin with C-models. Since each model has only one object in it, the predicates' adicies don't matter. For each predicate, there will only be two options: either the one thing satisfies it (by itself for one-placed predicates, or with itself i times for i-placed predicates), or it doesn't. So if $\mathcal{L}_M$ has n distinct predicates in it, there will be 2^n ways to make logically distinct C-models with just one object in the domain. In other words, C has 2^n singular isomorphism classes.

Now consider OPC-models. Once again, if $\mathcal{L}_P$ has m predicates, there will be 2^m ways to distribute that one object across its predicates. But suppose $\mathcal{L}_P$ has i quantifiers in it. For each quantifier, we can either put the thing in its domain or not. So there are 2^i possible distributions of the object across the quantifiers. But *one* of these distributions — the one where the object doesn't end up in the domain of *any* quantifier — is disallowed. That object has got to show up somewhere.[17] So there are $2^m(2^i - 1)$ ways to make logically distinct OPC-models with just one object in the domain. In other words, OPC has $2^m(2^i - 1)$ singular isomorphism classes.[18]

We proved in the last section that there was a one-to-one correspondence between these isomorphism classes; thus, there has to be just as may of the one kind as of the other. In other words, $2^n = 2^m(2^i - 1)$. But this equation only

[17]This is the reason, in fact, that in the completeness proof in §3.3, we had to tinker around with the sentence $\mathcal{Q}$. It was 'coding up', in the monistic language, the fact that the pluralist's quantifiers (which the 'Q_i's went proxy for) couldn't all be empty.

[18]We might suspect that if OPC were inclusive — that is, allowed for empty domains — this problem would be blocked. But that isn't right; if we didn't put the one object in one of the i quantifiers, then *the model wouldn't be singular*, which (by hypothesis) it is. Whether the model theory allows empty total domains doesn't come into it.

has an integer solution for n, m, and i when $n = m$ and $i = 1$. (Otherwise the right-hand side is odd, and greater than one, and so no power of two.) This means that $\mathcal{L}_P$ has only one quantifier — which means $\mathcal{L}_\text{P}$ isn't really a *pluralist* language after all! This completes the reductio when the background logic is OPC.

Next, OPC+-models. Here there are still 2^m ways to distribute one object across m predicates. But if $\mathcal{L}_\text{P}$ has i quantifiers, we have *exactly* i ways to put that one object into quantifier domains: we can put it in the first one, or put it in the second one, or ..., or put it in the ith one, and that's it.

This means that there will be $i(2^m)$ ways to make singular OPC+-models, in which case there will be $i(2^m)$ of the relevant isomorphism classes. Once again, the one-to-one correspondence tells us that there are just as many of these classes of OPC-models as of C-models. So $2^n = i(2^m)$. But, if $i \neq 1$, this means that $i = 2^j$ for some integer j. In other words: the OPC+-theorist is notation fodder only if she posits 2^j ways of being for some integer j.

Now consider OPS-models. If M is a model, let its *stamp* be the set of sorts of the language that have non-empty domains in M. If we're being inclusive or concessive, the singular models are all and only those that are isomorphic to one that has just one element, although that element can show up in several of its domains. In this case, if there are i quantifiers, then there are $2^i - 1$ ways to distribute one quantifier across i domains. So there are $2^i - 1$ stamps of singular models. If we're being exclusive, the singular models are all and only those that have all domains empty but for one, which has just one object in it. Here there are i stamps of singular models.

For each stamp, there will be a fixed number of interpretation functions the model can take, settled by facts about the languages' predicates. Let a predicate's *signature* be the set of sorts such that the predicate has at least one place restricted to that sort. If Π's signature is $\{1\}$, for instance, all of its predicate places are of sort 1; if it's signature is $\{2, 3, 5\}$, then all of its predicate places are either of sort 2, 3, or 5, and each of these sorts has at least one place in Π. If Π is a predicate with a signature S and T is a stamp, then say that Π *matches* T iff $S \subseteq T$.

For a given stamp T, let k_T the number of predicates that match T. (Note that, if no predicates match T, then $k_T = 0$.) For each stamp T, there will be exactly 2^{k_T} models with that stamp which can be made with just one element. (When $k_T = 0$, there is exactly one such model with that stamp; it's the one determined by a conjunction of existential statements for each quantifier saying either that the quantifier's domain is empty or that it has exactly one element in it.) As a result, where $\mathcal{T}$ is the set of stamps that a singular model can have, there will be $\sum_{T \in \mathcal{T}} 2^{k_T}$ isomorphism classes of singular models. As before, given the one-to-one correspondence discussed above, this means that

(19) $2^n = \sum_{T \in \mathcal{T}} 2^{k_T}.$

This places a significant constraint on the shape of the (sorted) pluralist's language: there are, in any intuitive sense, far fewer pluralist languages that satisfy it than there are that don't.[19]

If we're being exclusive, the right-hand-side of (19) will have only i terms in it, and each k_j will correspond to the set of predicates that can only take terms of a single sort. Call these terms *pure*. Pure predicates must be carefully aligned for notational defection to be a possibility. If there are two ways of being, for instance, there must be exactly the same number of pure predicates: assuming (without loss of generality) that $k_1 \leq k_2$, $2^n = 2_1^k + 2^{k_2}$, so $2^{n-k_1} = 1 + 2^{k_2-k_1}$. But this can happen only if $k_1 = k_2$. Likewise, if there are three modes of being, $2^n = 2_1^k + 2^{k_2} + 2^{k_3}$, so (assuming $k_1 \leq k_2 \leq k_3$) $2^{n-k_1} = 1 + 2^{k_2-k_1} + 2^{k_3-k_1}$. This can only happen if $k_2 = k_1$, in which case $2^{n-k_1} = 2 + 2^{k_3-k_1}$, so $2^{n-k_1-1} = 1 + 2^{k_3-k_1-1}$, which then requires that $k_3 = k_1 + 1$. In other words, a three-sorted pluralism can be only notational fodder if it has k pure predicates of two of its sorts and $k+1$ pure predicates for its third.

If we're being inclusive or concessive, we're even more highly constrained. Here we have $2^i - 1$ possible stamps. But the number of predicates that match each stamp cannot vary freely, because stamps are ordered by inclusion. For instance, if k_1 predicates match $\{1\}$ and k_2 match $\{2\}$, then there will be $k_1 + k_2 + j$ predicates that match $\{1, 2\}$, where j is the number of predicates of signature $\{1, 2\}$. As a result, applying the same reasoning as in the case of the three-sorted exclusive pluralist above, if we have two-sorted inclusive or concessive notation fodder, $k_1 = k_2$ and $k_1 + k_2 + j = k_1 + 1$, which means that $j = 1 - k_1$. So either $k_1 = k_2 = 1$ and $j = 0$, or $k_1 = k_2 = 0$ and $j = 1$. In other words, the only way we get a logic-preserving translation function between a two-sorted pluralist theory of this kind and a first-order theory is when the pluralist either has exactly two predicates, each of which takes only terms of a different sort, or exactly one predicate which takes terms of both sorts. The prospects for variance are limited indeed.

[19]Of course, there are infinitely many pluralist languages, so strictly speaking we can't compare the constraint-satisfiers to the non-constraint-satisfiers. But for any finite upper bound b on the size of the languages, there will always be far fewer languages with no more than b predicates that fail to satisfy the constraint than there are languages with no more than b predicates that satisfy it.

5 The Scorecard

Logical Realism provides a necessary condition for notational variance. The above result shows that, if notational variance is supposed to be motivated by worries particular to ontological pluralism (rather than a more general no-fact-of-the-matter-ism about metaphysics), this necessary condition is only met in a limited class of cases. For OPC, never; for OPC+, only when there are 2^j quantifiers; and for the various ways of counting on OPS, only when some more restrictive mathematical relations between ways of being and predicates are satisfied.

So when those conditions aren't met, we don't have notational variance. What about when the conditions *are*? There is room, of course, to insist that in those special cases where a logic-preserving translation scheme is possible, the theories in question are notational variants.

But there are two reasons to resist this conclusion. First: given how circumscribed the possibilities for variance are, making the variance charge stick seems to rely on some pretty circumstantial evidence.

Consider, for instance, an OPC+-endorsing, Aristotle-inspired pluralist who begins her career thinking that there are ten modes of being, corresponding to the classical ten non-overlapping categories: substance, quantitiy, quality, relation, place, time, position, state, action, and affection. The theory she endorses has ten primitive existential quantifiers, and so (since $10 \neq 2^j$ for integer j) cannot be a notational variant of a monist theory. But, thanks to learning a bit of physics, she decides that time and position really aren't separate categories but rather a species of, say, relation. She cuts the categories she believes in down to eight, so her revised pluralist theory uses eight primitive existential quantifiers.

Should we say that she used to have a legitimate dispute with her monist friends, but now their disagreement is merely notational? Our Aristotelian simply did not make the sort of change that should turn her from a thoroughgoing pluralist to someone who disagrees with the monist only notationally. The mere *number* of categories endorsed shouldn't tip the scales from 'legitimate disagreement' to 'notational variance'. Likewise, a (conciliatory) OPS-theorist with just three unary predicates — one restricted to abstracta and two restricted to concreta — shouldn't have her disagreement with a monist turn notational merely because she decides to give up one of her concreta predicates.

Second: if we insist that every pluralist theory that admits of a logic-preserving translation with a monist one is a notational variant of it, we get a strange result. Every monist theory (with at least one monadic predicate) will be translatable into some OPC+ theory that endorses unique sorting and has 2^j existential quantifiers. If we say all such pairs of theories are notational variants, we will have to say that every monist theory is a notational variant of some pluralist theory. In other words, there is no way it could be a fact of the matter that monism is true. On the other hand, it *could* be a fact of the matter that pluralism is true — it would be, for instance, if the true metaphysically perspicuous theory was an OPC+ one with three existential quantifiers in it.

This is really weird. If monism could be determinately false, then it should be possible for it to be determinately true, too. So this suggests we shouldn't think that a pluralist theory is a notational variant of a monist theory just because the math lines up right. We ought to regard monism and pluralism as generally different, and treat the existence of logical-truth-preserving translation schemes between certain pairs of them as a surprising oddity rather than a deep, revealing fact. But in this case, the worry has been resolved; pluralist theories won't generally be notational variants of monist ones, even given a unique sorting axiom, and there will be a fact of the matter as to whether monists or pluralists are right.

References

Alanen, Lilli (1994). "Sensory Ideas, Objective Reality, and Material Falsity." In John Cottingham (ed.), *Reason, Will, and Sensation: Studies in Descartes' Metaphysics*, 229–250. Oxford: Oxford University Press.

Chalmers, David, David Manley and Ryan Wasserman (eds.) (2009). *Metametaphysics*. Oxford: Oxford University Press.

Descartes, Rene (1984). "Author's Replies to the First Set of Objections." In John Cottingham, Robert Stoothoff and Dugald Murdoch (eds.), *The Philosophical Writings of Descartes*, volume 2, 74–92. Cambridge: Cambridge University Press.

Dorr, Cian (2005). "What We Disagree About when we Disagree About Ontology." In Mark E. Kalderon (ed.), *Fictionalism in Metaphysics*, 234–286. Oxford: Oxford University Press.

Eklund, Matti (2007). "The Picture of Reality as an Amorphous Lump." In Zimmerman et al. (2007), 382–396.

Enderton, Herbert B. (2001). *A Mathematical Introduction to Logic*. 2nd edition. Harcourt Academic Press.

Etchemendy, John (1983). "The Doctrine of Logic as Form." *Linguistics and Philosophy* 6(3): 319–334.

Field, Hartry (1991). "Metalogic and Modality." *Philosophical Studies* 62(1): 1–22.

Fine, Kit (2001). "The Question of Realism." *Philosophers' Imprint* 1(1): 1–30.

— (2005). "Tense and Reality." In *Modality and Tense*, 261–320. Oxford: Oxford University Press.

Hailperin, Theodore (1953). "Quantification Theory and Empty Individual-Domains." *The Journal of Symbolic Logic* 19(3): 197–200.

Harris, J. H. (1982). "What's So Logical About the Logical Axioms?" *Studia Logica* 41: 159–171.

Hawthorne, John (2006). "Plenitude, Convention, and Ontology." In *Metaphysical Essays*, 53–69. Oxford: Oxford University Press.

Hawthorne, John (O'Leary-) and Andrew Cortens (1995). "Towards Ontological Nihilism." *Philosophical Studies* 79(2): 143–165.

Hirsch, Eli (2002). "Quantifier Variance and Realism." *Philosophical Issues* 12, *Realism and Relativism*: 51–73.

— (2007). "Ontological Arguments: Interpretive Charity and Quantifier Variance." In Zimmerman et al. (2007), 367–381.

Hoffman, Paul (1996). "Descartes on Misrepresentation." *Journal of the History of Philosophy* 34(3): 357–381.

Kim, Jaegwon (1993). *Supervenience and Mind*. Cambridge: Cambridge University Press.

Kreisel, Georg (1967). "Informal Rigor and Completeness Proofs." In Imre Lakatos (ed.), *Problems in the Philosophy of Mathematics*. Amsterdam: North-Holland.

Lambert, Karel (2001). "Free Logics." In Lou Goble (ed.), *The Blackwell Guide to Philosophical Logic*, 258–279. Malden, Mass.: Blackwell.

Leblanc, Hugues and R. H. Thomasson (1982). "Completeness Theorems for Some Presupposition-Free Logics." In *Existence, Truth and Provability*. Albany, NY: SUNY Press.

Lewis, David (1983). "New Work for a Theory of Universals." *The Australasian Journal of Philosophy* 61: 343–377. Reprinted in Lewis (1999): 8–55.

— (1984). "Putnam's Paradox." *The Australasian Journal of Philosophy* 62: 221–236. Reprinted in Lewis (1999): 56–60.

— (1986). *On the Plurality of Worlds*. Malden, Mass.: Blackwell.

— (1999). *Papers in Metaphysics and Epistemology*. Cambridge: Cambridge University Press.

Loux, Michael J. and Dean W. Zimmerman (eds.) (2003). *The Oxford Handbook of Metaphysics*. Oxford University Press.

Ludlow, Peter (1999). *Semantics, Tense, and Time*. Cambridge, Mass.: MIT Press.

McDaniel, Kris (2009). "Ways of Being." In Chalmers et al. (2009).

— (forthcoming). "A Return to the Analogy of Being." *Philosophy and Phenomenological Research* .

McGee, Vann (2000). "Everything." In Gila Sher and Richard Tieszen (eds.), *Between Logic and Intuition*, 54–78. Cambridge: Cambridge University Press.

— (2006). "A Rule for Everything." In Rayo and Uzquiano (2006), 179–202.

Mellor, D. H. (1981). *Real Time*. Cambridge: Cambridge University Press.

— (1998). *Real Time II*. London: Routledge.

Normore, Calvin (1986). "Meaning and Objective Being: Descartes and His Sources." In Amélie O. Rorty (ed.), *Essays on Descartes' Meditations*, 223–241. University of California Press.

Plantinga, Alvin (1987). "Two Concepts of Modality: Modal Realism and Modal Reductionism." *Philosophical Perspectives* 1: 189–231.

Prior, A. N. (1968). *Papers on Time and Tense*. Oxford: Oxford University Press.

— (1977). *Worlds, Times, and Selves*. Ed. Kit Fine. London: Duckworth.

Putnam, Hilary (1987*a*). *The Many Faces of Realism*. La Salle, Ill.: Open Court Press.

— (1987*b*). "Truth and Convention: On Davidson's Refutation of Conceptual Relativism." *Dialectica* 41: 69–77.

Rayo, Agustín and Gabriel Uzquiano (eds.) (2006). *Absolute Generality*. Oxford: Oxford University Press.

Restall, Greg (2005). "Multiple Conclusions." In Petr Hajek, Luis Valdes-Villanueva and Dag Westerstahl (eds.), *Logic, Methodology and Philosophy of Science: Proceedings of the Twelfth International Congress*, 189–205. London: Kings' College Publications.

Sider, Theodore (2001). *Four Dimensionalism*. Oxford University Press.

— (2003). "Reductive Theories of Modality." In Loux and Zimmerman (2003), 180–208.

— (2009). "Ontological Realism." In Chalmers et al. (2009).

Sosa, Ernest (1993). "Putnam's Pragmatic Realism." *The Journal of Philosophy* 90(12): 605–626.

Szabó, Zoltán Gendler (2003). "Nominalism." In Loux and Zimmerman (2003), 11–45.

Turner, Jason (2010). "Ontological Pluralism." *The Journal of Philosophy* 107(1): 5–34.

Williamson, Timothy (1988). "Equivocation and Existence." *Proceedings of the Aristotelian Society* 88: 109–127.

— (2006). "Absolute Identity and Absolute Generality." In Rayo and Uzquiano (2006), 369–389.

Zimmerman, Dean, John Hawthorne and Theodore Sider (eds.) (2007). *Contemporary Debates in Metaphysics*. Malden, Mass.: Blackwell.