The Construction of Logical Space and the Structure of Facts

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0 Getting Started

In *The Construction of Logical Space*, Agustín Rayo (2013) defends *trivialism*, according to which mathematical statements are, in some sense, trivial. In what sense? Well, the basic idea is that truths about the mathematical realm don't really go above and beyond the non-mathematical realm. Mathematical statements are disguised ways of talking about the non-mathematical world.

How does it work? Well, consider the statements:

(1) My hand has five fingers.

(2) The number of fingers on my hand is five.

Rayo's trivialism, as it applies to these claims, can be expressed in a number of ways, all of which are supposed to be equivalent. Here are three:

- For (1) to be the case *just is* for (2) to be the case.

- Claims (1) and (2) have the same truth-conditions; that is, they make the same demands of a world.

- Claims (1) and (2) are different ways of expressing the same fact.

For concreteness, I'm going to focus on the third.

Resistance to Rayo's trivialism comes from a view he calls *metaphysicalism*, which for our purposes we can think of coming in two parts. The first, metaphysical part says that facts come pre-equipped with a unique structure. The second, semantic part says that a sentence can only express a fact if its subsentential bits correspond one-to-one with the elements of that structure. From this perspective, a given fact F can either have the structure of a monadic predication, predicating 'having five fingers' of my hand, or it can have a more complex structure predicating a relation between the number five and the fingers on my hand, but it can't have both. So if any fact has the right structure to be expressed by (1), it can't be expressed by (2), and vice versa.

One line of resistance to metaphysicalism — which Rayo treats with approval (15-17, 70-78)[1] — stems from broadly Fregean thoughts about stipulation. Suppose we started by speaking a language that didn't let us talk about numbers, but did let us form 'counting sentences'. In that language, (1), for instance, would be true. Then it seems we should be able to *explicitly stipulate* Hume's principle,

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[1] Date-free page numbers reference Rayo 2013.

(HP) The number of *F*s = *n* iff there are *n* *F*s,

and thereby give content to number talk. The idea is that we would thereby explicitly make (2) and its ilk pick out the same facts that (1) and its ilk already pick out. The rest of the number-involving sentences would end up labeling whatever facts they would need to label to make sure our quantifiers and so on kept validating the inferential behavior we expect for them. If these stipulations are kosher, then metaphysicalism can't be right, and the main bar to Rayo's trivialism is removed.

On the face of it, this might work in either of two ways. On a *deflationary* picture, in and of themselves facts simply have no structure. Since they are structureless, we are free to 'carve' them in whatever way we please. This 'carving' is done by our linguistic practices. Once our linguistic practices are up and running, they impart structure to the facts; but in the order of explanation, our linguistic practices come first.

Note that deflationists can say not only that facts have structure, but that they had structure before we came along, and that they would have had structure even if we hadn't been around. But they can't *mean* it. That's because the deflationist can think that the way we carve the facts projects modally and temporally, infecting the modal and temporal claims we make. But this projection shouldn't be taken too seriously; it's a feature of how we talk, and ignores the important explanatory relation between facts' structure and our practices.

The alternative *inflationary* picture holds that facts do have structure in and of themselves, which sentences need to match — it's just that they have a *lot* of it. It's a mistake to talk about *the* structure that facts have. Facts are *multiply structured*. For instance, quite independent of any linguistic behavior on our part, there is a fact that has *both* the structure *Fa and* the structure *Rab*. Since one fact has each of these structures, it's an apt candidate to be expressed by sentence (1) and also sentence (2). We don't carve facts with our linguistic practices; rather, we pick them out via one of the many, many structures they come pre-equipped with.

As I read *The Construction of Logical Space*, it sits best with the deflationary picture. I'll offer three reasons for thinking so before going on to complain a bit about that picture.

1 INFLATIONISM

First, trivialism is supposed to protect us from a certain sort of mistake. Traditional non-trivialists about mathematics, whether they are platonists or nominalists, think that there is a certain sort of *risk* in moving between (1) and (2), and there's a certain sort of risk in stipulating (HP): If the universe doesn't play ball by providing a domain of numbers, (1) may be true but (2) cannot be. The risk arises from it being a substantive question whether the world plays ball or not.

If facts are multiply structured, though, we get a different kind of risk when moving between (1) and (2) or stipulating (HP): The risk that, even though the facts have many structures, *none* of those structures corresponds to anything like (2). Inflationism doesn't seem to remove the risk. By contrast, deflationism does: if a fact has a number-involving structure in virtue of us using a certain sentence to get at it, then there's no risk that the fact fail to have the structure needed to make (HP) true. The risk goes away because our stipulation of (HP) *gives* the fact its structure. Once we stipulate (HP), there's no further condition any fact needs to meet in order to make sure our stipulation works.

Second, I have a hard time squaring the inflationist picture with what Rayo says about the 'Julius Caesar problem'. The thought behind that problem, which goes back to Frege, runs like this: Suppose we stipulate (HP) and thereby introduce numerical terms. We can now form the sentence

    (3) Julius Caesar = the number 7.

But is (3) true, or false? It seems that nothing about our stipulation settles this, and it's difficult to see what *else* could settle it.[2]

Rayo's own view is that (3) simply has no truth-value — presumably, that is equivalent to there being no fact that (3) expresses. Discussing this view, he writes:

> Metaphysicalists will claim that something important has been left out. For in the absence of well-defined truth-conditions for [3], it is unclear which of the objects carved out by the metaphysical structure of reality has been paired with '[the number 7]'. But [I] disagree: it is simply a mistake to think that such pairings are necessary to render a singular term meaningful. In fact, once should *expect* there to be mixed identity statements that lack well-defined truth-conditions. For when a sentence has no clear role to play in communication — as is the case of [3], philosophical discussion aside — linguistic practice generates no pressure for it to be assigned truth-conditions even when its constituent terms figure meaningfully in other sentences. (80-81)

I can make sense of this if Rayo is a deflationist; facts only have the structure they have thanks to our linguistic practices, so if our linguistic practices don't pick out a fact for (3) to express, there is no such fact. But I don't know how to square this thought with inflationism. What structure facts have is independent of our linguistic practices. One fact has a structure involving 7, and another has a structure involving Julius Ceasar. (These facts might each have *other*, non-7- or non-Ceasar-involving structures, but that's neither here nor there right now.) Why *shouldn't* there be an identity fact or a distinctness fact involving both?

If facts have their structure 'on their own', independent of us and our interests, then it is natural to expect them to obey a kind of combinatorial principle.

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[2]*The Foundations of Arithmetic*, §§56, 66; see Heck 1997 for historical discussion.

To fix ideas, suppose facts are *immanent*: there is no such thing as a 'false,' or non-obtaining, fact. Then it is natural to expect:

(C) If $F_1, \ldots, F_n, G$ are facts that have structures involving $x_1, \ldots, x_n$ and the n-adic relation R, respectively, then either some fact has the structure $Rx_1 \ldots x_n$ or some fact has the structure $\sim Rx_1 \ldots x_n$.

(C)'s failure would be very odd, to say the least, and would deserve explanation. The deflationist has a simple explanation: Facts have whatever structure our linguistic practices impart to them, and no more. (C) fails because our linguistic practices don't assign structure to facts in a way that vindicates it — end of story. But I have no idea what explanation the inflationist has to offer of the failure.

Finally, Rayo's take on absolutely unrestricted quantification — on whether there is, as he puts it, a 'maxi-domain' — doesn't sit well with inflationism either. He writes,

> ... the existence of a maxi-domain would require a final answer to the question of what counts as a possible system of compositional representation. And I see no *prima facie* reason to think that the notion of representation (and our notion of linguistic representation, in particular) are constrained enough for this question to have a definite answer... [I]t is hard to say in advance what would count as a possible compositional language. Whenever we dream up a new mechanism for representing reality, the potential for a new compositional language — and hence a new way of carving up the world — will be in place. (29)

The idea, if I understand it, is that there can be no once-and-for-all domain unless there is a once-and-for-all fact about what languages are possible, and Rayo doubts there is any such fact.

As I presented them, both inflationism and deflationism accept

(R→S) If there is a linguistic representation of a fact F, then that fact has a structure isomorphic to that representation.

They simply disagree about the order of explanation: The deflationist thinks the structure is explained by the representation, whereas the inflationist thinks the representation is (partially) explained by the structure. The complaint against unrestricted quantification seems to rely on the converse, though:

(S→R) If F is a fact with a structure S, then there is some (possible) linguistic representation of F that has a syntactic structure isomorphic to S.

The objection to unrestricted quantification moves from indefiniteness about the consequent in (S→R) to indefiniteness about the antecedent.

I can see why the deflationist would accept (S→R): facts 'get' their structure from representations, so any structure a fact has must at least be in principle linguistically representable. But I don't see why an *inflationist* should accept this. The facts just have whatever structure they have — why be so sure that *every* structure a fact have be linguistically representable? In particular, if there is indefiniteness in what counts as a linguistic representation, why think that it makes for indefiniteness in what structure facts have, rather than indefiniteness in which of a fact's many structures is linguistically representable?

2   DEFLATIONISM

All this makes me read Rayo as a deflationist. But I have a worry about deflationism. To see the worry, let me start with what I see as a *virtue* of Rayo's main foil, metaphysicalism.

Here are two puzzles of intentionality. A first one troubles us still: How do our minds reach out and 'grasp,' in thought, the material, non-mental world? We seem to have forgotten the second one, though in various forms it bothered philosophers from Parmenides to Russell: How is it that I can have *false* thoughts? If London is pretty, how can I *think* to myself 'London is ugly,' when there is no ugliness-of-London for me to represent?

Metaphysicalism has a nice, unifying explanation for these puzzles. Suppose, to fix ideas, that we think in a language of thought. (The assumption is dispensable, but my point would take a lot more time to make without it.) First, we get a correlation between mental words (concepts) and parts of the world (objects and properties) through causal interaction with those parts of the world. Once we've done that we can represent to ourselves ways the world is not by recombining those mental words in certain ways.

According to metaphysicalism, facts have distinguished structure. Our concepts latch on to nodes in this structure. We then recombine those concepts syntactically; if there is a fact with that structure, then our new thought is true. Otherwise, it is false.

It's important for this picture that the structure *come first*. The representation depends on the structure, so the structure can't depend on the representation. Since the inflationist agrees that structure precedes representation, she can accept this unifying explanation of intentionality too. But the deflationist can't. The structure *isn't there* until the linguistic representations have been established; but then those representations cannot themselves depend on correlations between concepts and nodes in the structure.

Here's a just-so story to see the worry. Suppose that, in the beginning, facts were unstructured. In the Garden of Eden, Adam came across some unstructured facts and decided to give them names. He names *this* one 'Elly the elephant is big,' and he names *that* one 'Tony the tiger is drinking at the pool,' and so on. Then all the existing facts are thereby given a structure. But nothing

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in Adam's labeling behavior tells us how to *extend* that structure. Suppose, for instance, that Adam labels these two facts before Elly's first drink at the pool. When Elly starts drinking, a new fact comes into existence; and it seems pretty clear that Adam's hand should now be forced. Given the labels he's already assigned the first two facts, he needs to call this third one 'Elly the elephant is drinking at the pool'. But if facts are unstructured, then *before* Adam gives this new fact a label there's no real sense in which it 'shares' any structure with these other two facts, and so no explanation as to why Adam's hand should be so forced.

This point is easy to miss. Rayo talks quite a lot about 'truth-conditions,' and assigning truth-conditions to sentences. But nowhere in the book do we get a *metaphysics* of truth-conditions. We're told how to 'specify' them, but the specification is of course always simply in terms of other sentences, which in turn need truth-conditions specified for them, and so on down the line. There's no story as to how the linguistic rubber meets the extra-linguistic road, to get the whole enterprise running down the track. Without that, it is difficult to see what resources the picture has left to solve these puzzles of intentionality.

3 MODERATE METAPHYSICALISM

Unlike the deflationist, the inflatonist has an explanation of intentionality because it puts fact-structure first. It differs from metaphysicalism in that it denies that a fact's structure is *unique*; but the structure is *there*, and can be appealed to in these explanations.

Rayo considers another view, which he calls *moderate metaphysicalism* (11-13), according to which (i) facts *do* have a unique, metaphysically privileged structure; but (ii) sentences need not mirror that structure in order to be meaningful. As I understand it, the view is a sort of hybrid between inflationism and deflationism: Facts have *one* structure which is 'really there,' independent of linguistic practice, but can have others imparted to them by further linguistic stipulations.

On the face of it, the moderate metaphysicalist can perhaps get the best of both worlds. For example, she can hold that our talk of fingers and hands and so on latches on to this privileged metaphysical structure, adapting the metaphysicalist's explanation of intentionality for that type of talk. She can then go on and stipulate (HP) to give a bunch of facts some new linguistic representations, ones which *don't* match their metaphysical structure. Any further new structure they get is then of the deflationary kind, and she can say the same things that Rayo does about, e.g., the Ceasar problem or absolutely unrestricted quantification.

On this picture intentionality is explained in two steps. First, there is a *core language*, which we'll call CORE, where the subsentential bits match parts of facts' metaphysical structures, and sentences are true if and only if they corre-

spond to a fact with an isomorphic metaphysical structure. The CORE sentences mean what they do for just the reasons the metaphysicalist says. Then there are some linguistic stipulations, THE STIPULATIONS, with combine with the truths in CORE to produce a larger language, which we'll call BIG. The meanings of the sentences of BIG are explained by appeal to (i) THE STIPULATIONS plus (ii) the meanings of the CORE sentences that THE STIPULATIONS pair them with.[3] Call this the *Two-Stage* Picture of Intentionality.

Although he is officially neutral about moderate metaphysicalism (while expressing some skepticism), Rayo suggests that everything in his book is consistent with it (12). So maybe he should endorse the Two-Stage Picture.

But I'm not so sure that would be a good idea given the sort of philosophy of mathematics Rayo wants to endorse. Note first that, if the existence of mathematical entities is truly *trivial*, then presumably mathematical entities shouldn't show up in the unique, metaphysically privileged structure of facts. (Otherwise it's hard to see what's so trivial about them, as they would seem to make up some of the world's basic building blocks.) On the Two Stage Picture, this means that the CORE language should be mathematics-free.

As Rayo argues towards the end of the book, stipulations are guaranteed to succeed so long as the stipulated theory is *conservative* (185-187): so long as the new theory, combined with the old theory, doesn't take any invalid arguments in the old language and turn them valid. As Hartry Field noted a long time ago, it's a pretty good bet that most mathematical theories are conservative, so that's a low bar (Field 1980: 12–14). So we can add mathematics to the CORE language, no problem.

But, as Field *also* noted, a difficult question remains. Take our best theory, and call it BEST. It's shot through with mathematics. Since it's our best theory, we ought to believe it. If we are to use the Two-Stage Picture, we had better be able to get to BEST by starting from our mathematics-free CORE and stipulating some further mathematics. In the lingo, CORE needs to be a *nominalistic subtheory* of BEST.

It's a very difficult question as to whether there is *any* nominalistic subtheory of BEST. That's what Field's *Science Without Numbers* project was trying to determine. So the prospects for a defense of trivialism from the Two-Stage Picture are held hostage to the results of Field's project. Trivialism, I thought, was supposed to *remove* the need for de-mathematizing science. But I can't see any way for it to remove that need while still hanging on to the ideas that made inflationism attractive in the first place.

4 CONCLUSION

Where does this leave us? A full-fledged, deflationary trivialism takes away metaphysicalist's nice resolution of the puzzles of intensionality. The move to

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[3]Whether directly or indirectly.

moderate metaphysicalism and the Two State Strategy gives us back our resolution, but at the cost of making the triviality of mathematics hostage to the nominalization of science. For my part, I prefer the frying pan of the nominalization project to the fire of mysterious intensionality, but prospective trivialists will have to choose for themselves.

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